Journal of Sustainable Development in Africa (Volume 13, No.4, 2011)
ISSN: 1520-5509
Clarion University of Pennsylvania, Clarion, Pennsylvania

A CARPENTRY CUTTING STOCK PROBLEM: A CASE STUDY FOR PLANKS CUTTING IN ZIMBABWE

Kazunga Cathrine, Mutambara H. N. Lillian and Mapurisa Jabulani
Bindura University of Science Education

ABSTRACT

The carpentry sector like any other industry is faced with a cutting stock problem to minimize incurred waste. The main purpose of this project was to develop a mathematical model which will solve the cutting stock problem using column generation approach for Ashtons Company in Chinhoyi. The interview method was used to collect data relating to the cutting stock problem. The column generation approach of iterative computational routines was used because it develops successively better solutions until an optimal solution is obtained. The results revealed that the method is an appropriate method in solving business problems, that is, how many boards should be cut to meet demand with minimum incurred waste. A user friendly graphical user interface was developed using Visual Basic programming which could be used by the carpentry manager.

Key words: cutting stock problem, feasible solution, optimal solution, integer programming

INTRODUCTION

The cutting stock problem arises from many physical applications. Imagine that you work in a carpentry department, and you are the manager in the cutting stock division. We have a number of very long pieces of wood of fixed length waiting to be cut, yet different customers want different numbers of pieces of wood of various sized length. How are you going to cut the pieces of wood so that the least amount of turnovers is wasted? This turnout to be an optimization problem which is an integer programming (IP) problem. In this study we are going to develop an algorithm for carpentry cutting stock problem that employ column generation approach to obtain an optimal integer solution. We formulate a branching rule that can be incorporated into the sub problem to allow column generation at each node in the branch-and-bound tree.

When small items are being cut out from large objects, two problems arise. The first one is the assortment problem addressing the issue of choosing proper dimensions for the large objects. The second one is the trim loss problem addressing the issue of how to cut out the small items from the given large objects in such a way that waste material will be minimized, (Hinxman, 1979).

In practice, the small items are known as order list and the large objects are known as stock material. Especially in this project and other industries, the small items are called the product piece of wood (in this case are boards of length 75cm, 125cm, and 225cm) while the large objects are called the raw pieces of wood (the board of length 425cm). In the cutting process the stock material can seldom be used as a whole, some residual pieces or trim loss will be produced. Since the primary objective of the cutting process is to minimize the wastage the problem is known as the trim loss problem, (Dyckhoff, 1990). The combination of the assortment problem and the trim loss problem is known as the cutting stock problem (CSP).

The CSP arises in many mass production industries where large stock sheets or reels must be cut into smaller pieces. Usually stock materials are pulp and paper, steel, glass, wood, plastic and textiles, (Dyckhoff, 1990). This study concentrates on the CSP in the carpentry department which have wood as the stock material. The problem studied is a real life problem occurring at Ashtons Company in Chinhoyi. The company started selling the pieces of wood from 2000. Due to the current harsh economic conditions prevailing in Zimbabwe, there is need to plan and come with the best outcome out of the few resources available.

In production industry, stock material (pieces of wood) need to be cut properly to avoid a situation where organisation cease to provide viable goods and services due to poor resource management. The researcher liased with a carpentry department of Ashtons in Chinhoyi and realise that, despite current Zimbabwe economic conditions, the cutting of pieces of wood (board) incurred much waste. The boards were cut using traditional ways which has a lot of waste incurred. The wastage might lead to increase in cutting down of trees to meet the demand, which might lead to deforestation. The thrust of this project is to apply the column generation approach to determine the optimal number of boards to be cut to meet demand with minimal waste incurred. The method can be applied in different sectors of the economy, in industry, banking, engineering, education, or even in households.

Since each decision problem is characteristically complex and can have a significant impact on the health of a firm, it is almost impossible for any decision maker to intuitively take full account of all the factors impinging on a decision simultaneously. Therefore, it becomes useful to find some method of separating such decision problems into parts in a way that would allow a decision maker to think through the implications of each set of factors at a time in a rational, consistent manner, (Winston, 1994). The study comes in to provide the tools for decision making for the operation manager of Ashtons

LITERATURE REVIEW

THE CUTTING STOCK

The very first formulation of the CSP was produced by Kantorovich in 1930, although it was published in English only in 1960, (Kantorovich, 1960). Scientific research started about forty five years ago and fast growing numbers of different formulations solution methods for the CSP have been presented since
1930, (Karelahti, 2002).

In the following paragraph we define some important characteristics of the CSP. CSP can be efficiently classified by using the classified scheme developed by Dyckhoff (1990). Since there is a strong relationship between cutting and packing, the scheme applies to both problem categories. The relationship results from the duality of material and space, the duality of solid material body and the space occupied by it. In certain sense, cutting can be seen as the space occupied by small object into the space occupied by large objects. On the other hand, packing can be seen as cutting the empty space of the large objects into parts of empty space some of which are occupied by small items, the other being trim loss, (Loren, 2000). The cutting stock problem can be described as follows:

Assume d_i is the demand for product i. The stock consists of pieces of wood that can be cut in pattern or combination j. The possible patterns are denoted by a_i; j being the number of products of length i that are the number of applying pattern j. The number of times pattern or combination j will be used is x_j . The cutting pattern can now be formulated as the Mixed Integer Programming Model, Karelahti (2002).

THE COLUMN GENERATION APPROACH

Column generation approach is a powerful tool for solving large linear programming problem (LPP). Such

Linear programming may arise when the columns in the problem are not known in advance and complete enumeration of all columns is not an option. It appears this approach has been successfully explored in several other applications, such as the well known cutting stock problem, vehicle routing and crew scheduling, (Haessler, 2002).

When a linear program has many variables, it is very time consuming to price out each non-basic variable individually. The column generation approach come into play to price out each non-basic variable, and tell which variable should enter the basis. It can be used to increase the efficiency of the revised simplex algorithm.

Winston (1994) used column generation approach to solve the CSP for Woodco and the minimum waste incurred was only 15 feet. The knapsack sub problem as solved using branch-and-bound procedure, the master problem was solved by an advanced method of simplex method called the product form of the inverse. Winston (1994) also cited that if we have a starting basic feasible solution for a CSP, we need not to list all possible ways which a board may be cut. For each iteration, a good combination (one that will improve the z-value when entered into the basis) is generated by solving a branch-and-bound problem. The fact that we do not have to list all the ways a board can be cut is very helpful, a CSP that was solved by Gilmore and Gomory (1961) cited in Winston (1994) for which customers demanded boards of 40 different length involved over 100 million possible ways a board could be cut. At the last stage of the column generation procedure of this problem, solving that single branch-and-bound problem indicated that none of the boards can be cut. At the last stage of the column generation procedure of this problem, solving that single branch-and-bound problem indicated that none of the 100 million (non-basic ways) would price out favorably. This method is certainly more pleasant than using, the 1 by 3 column vector, cBasic Variable B_1 in revised simplex algorithm to price out all 100 million variables. The column generation saves time. Thus, this motivated the researcher to use the column generation approach to solve the CSP for Ashtons Company. Kalvelagen (2002) in his paper describes an implementation of the column generation algorithm using GAMS. The well known cutting stock problem was used. The algorithm consists of 2 different models, a master problem and sub-problem which exchange information. A mixed integer problem for this problem was trivially formulated in GAMS once they have enumerated all possible cutting patterns. Even for this extreme problem with 4 lengths they have 17 possible patterns or combinations. They manage to minimize the waste incurred. The review of branch and bound procedure and the product form of the inverse will be considered since they are going to be used in solving the LPP of the CSP.

METHODOLOGY AND MATHEMATICAL FORMULATION

PROBLEM STATEMENT

The researcher interviewed the operations manager of carpentry department (which is one of the four departments). The interview showed that the Ashtons Company from the previous sales in carpentry department is expecting to cut 20 boards of length 425cm. These boards will be cut such that they get 25 boards of 75cm, 25 boards of 75cm and 15 boards of 225cm. But does this combination of cutting the boards of length 425 optimal? Does it give minimum incurred waste? Ashtons Company sells 75cm, 125cm and 225cm pieces of wood (boards).

The operations manager of carpentry department, who must meet demand by cutting up 425cm boards, wants to minimize (min) the waste incurred to meet the demand for a week. The operations manager of carpentry department must decide how each 425cm board should be cut. Hence, each decision corresponds to a way in which a 425cm board can be cut. Many ways of cutting a board need not to be considered. For example it would be unwise to cut a board into one 225cm, and one 125cm, we could just easily cut the board into a 225cm piece, 125cm piece and a 75cm piece. Any pattern that leaves 75cm or more of the waste need not be considered because we could use the waste to obtain one or more 75cm board. The Table 1 shows the different ways for cut a board of 425cm in the cutting stock problem.

Table1: Ways to cut a board of 425cm in the cutting stock problem

Combination	Number of 75cm boards	Number of 125cm boards	Number of 225cm boards	Waste in cm
1	5	0	0	50
2	4	1	0	0
3	2	2	0	25
4	2	0	1	50
5	1	1	1	0
6	0	3	0	50

Decision variables:

Let x_i = number of 425cm boards cut according to combination i

Thus,

x_1 = number of 425cm boards cut according to combination 1.

x_2 = number of 425cm boards cut according to combination 2.

x_3 = number of 425cm boards cut according to combination 3.

x_4 = number of 425cm boards cut according to combination 4.

x_5 = number of 425cm boards cut according to combination 5.

x_6 = number of 425cm boards cut according to combination 6.

MATHEMATICAL MODEL

Using information on Table 1 to formulate the LPP:

Ashtons waste + total customer demand = total length of boards cut

Total customer demand = 25(75) + 20(125) + 15(225)

Thus; the total customer demand = 7750cm

Total length of the boards cut = $425(x_1 + x_2 + x_3 + x_4 + x_5 + x_6)$

Ashtons waste (in cm) = $425x_1+425x_2+425x_3+425x_4+425x_5+425x_6 -7750$

Ashtons' objective function is to minimize incurred waste:

$$\min z = 425x_1 + 425x_2 + 425x_3 + 425x_4 + 425x_5 + 425x_6 - 7750 \quad (1)$$

This means that Ashtons carpentry department can minimize its waste by minimizing the number of 425cm of must be cut.

Ashtons faces the following constraints:

Constraint 1: At least 25 boards of length 75cm to be cut

Constraint 2: At least 20 boards of length 125 to be cut

Constraint 3: At least 15 boards of length 225cm to be cut

Constraint 1 becomes:

$5x_1 + x_2 + 2x_3 + 2x_4 + x_5 > 25$

Constraint 2 becomes:

$x_2 + 2x_3 + x_5 + 3x_6 > 20$

Constraint 3 becomes:

$x_4 + x_5 > 15$

The coefficient of x_i in the constraint for k cm boards is just the number of k cm boards yielded if a board is cut according to combination i. It is clear that xi should assume integer values.

The linear programming problem is as follows:

$\min z = 425x_1 + 425x_2 + 425x_3 + 425x_4 + 425x_5 + 425x_6 – 7750$

such that

$5x_1 + x_2 + 2x_3 + 2x_4 + x_5 > 25$ (board of length 75cm constraint)

$x_2 + 2x_3 + x_5 + 3x_6 > 20$ (board of length 125cm constraint)

$x_4 + x_5 > 15$ (board of length 225cm constraint)

$x_i > 0$ where $i = 1; 2;…; 6$

SOLUTION METHOD

The branch-and-bound procedure was used when solving the CSP using column generation approach. Therefore, the optimal solution to Ashtons' cutting stock problem is given by:

$x_2 =5/2$, $x_6 =5/6$ and $x_5 = 15$

We could obtain a "reasonable" integer solution by rounding x6 upwards. This will yield the integer solution:
$x_2 = 3$, $x_6 = 1$ and $x_5 = 15$

Substituting the values of x_1, x_2, x_3, x_4, x_5 and x_6 from column generation approach into objective function (1) gives:

$$
\begin{aligned}
z &= 425(0) + 425(3) + 425(0) + 425(0) + 425(15) + 425(1) -750 \\
&=8075 -7750 \\
&= 325
\end{aligned}
$$

Waste incurred using column generation approach is 325cm. The wasted incurred by the current way of cutting the pieces of wood = 20(425) -7750 = 750cm The carpentry department is currently incurring a waste of 750cm after cutting 20 pieces of wood of length 425cm for the week. After working out the CSP, the recommended number of pieces of wood of length 425cm to be cut is 19. The department will save a waste of 750cm 325cm = 425cm. They will have 2 extra pieces of wood of length 75cm and 1 extra sheet pieces of wood of length 125cm after meeting the demand of the week. Save 56 percent of the waste incurred from the traditional method.

GRAPHICAL USER INTERFACE (GUI)

A GUI was developed for the CSP of Ashtons. It produces an automated CSP because of the advantage of GUI which are listed below. GUI can be used because it is user friendly, faster for single operation, self teaching and experimentation is easier and again good at multi media.

The GUI for the CSP was developed using Visual Basic programming. Visual Basic 6.0 is a version of BASIC released in 1991 by the Microsoft Corporation to allow easy, visual oriented development of Windows application. Programs such as Visual Basic 6.0, which are designed by Microsoft Windows, are easy to use once you learn a little jargon and few basic techniques. Although Windows may seem intimidating if you have never used it before, you need to learn only a few basic techniques, (Schneider, 2004). A pseudo code was developed using binary search and linear search. It was used for programming to come up with the source code. The pseudo code is shown below:

```
//a, b, c are quantities of planks (pieces of wood)//
Initialize a, b, and c
If a < b < c
P = the number of 75cm planks
Repeat until P ends
Cut all sizes on one plank
Loop
While b <> end
do
Cut 125cm and 225cm planks
Loop
25
Cut 225cm
read all planks
End if
If a < b > c and a < c
Repeat until P ends
Cut all sizes on one plank
Loop
While c <> end
do cut 125cm and 225cm planks
Loop
do until b = end
cut 125cm
Loop
Read all planks
```

end if

If a > b < c

Repeat until c ends

Cut all sizes on one plank

Loop

While b <> end

do cut 4 by 75cm planks and a 125 cm plank Loop

While a <> end

Cut 75cm planks

Loop

Read all planks

if a > b < c and a > c

Repeat until b ends

Cut all sizes one plank

Loop

While c ends

do cut 75cm and 225cm

Loop

While a is not equal to 0

Repeat

Cut 75cm planks only

Loop

Read all planks

End if

The pseudo code was developed into a source code which produces the GUI.

The GUI was developed because the operations manager will not be able to use the column generation approach for other orders in future. This GUI will be the best decision tool for CSP of Ashtons. The GUI which is shown on Fig 3.1 below:

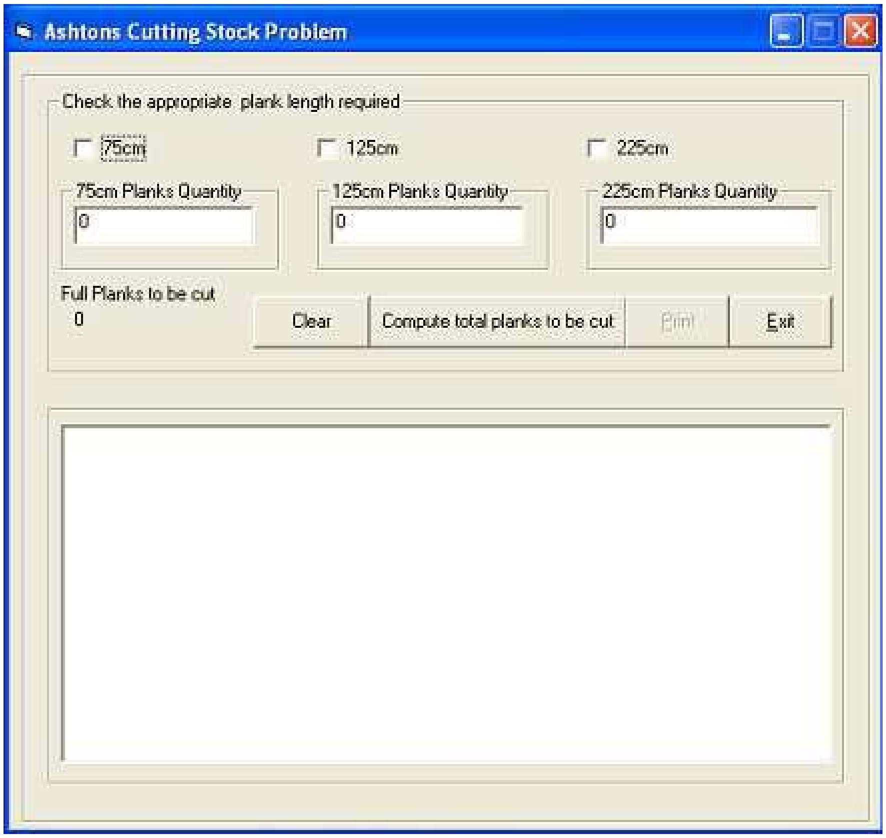

Figure 1: Graphical User Interface for cutting stock problem designed with Visual Basic 6.0.

The GUI can be learnt in a short time and be used at the Ashtons Company. The researcher use Visual Basic because it provide the tools one need to create window with familiar elements like check boxes, text boxes, command buttons, list box The carpentry department manager can also use the graphical user interface to find the number of pieces of wood of length 425cm to be cut and the combinations for cutting from a given order. The combinations can be printed and be used by the people who work in the workshop when cutting the pieces of wood of length 425cm to meet the demand for each order incurring minimum waste. The display will also show the pieces of wood to be stored in the stock for future use. The operations manager was impressed by the use of the GUI which makes his work easier. For example, if the operations manager has the following demand for any other week; 20 pieces of wood of size 75cm, 15 pieces of wood of size 125cm, and 10 pieces of size of wood of 225cm, he will click on all the check boxes and enter 20 on the first text box, 15 on the second and 10 on the third. He then clicks the compute button. The output on Figure 2 will be displayed in the list box. The output will be printed using the print button and then sent to the workshop and menus from the toolbox. Example of how it is used is displayed fig 2 below.

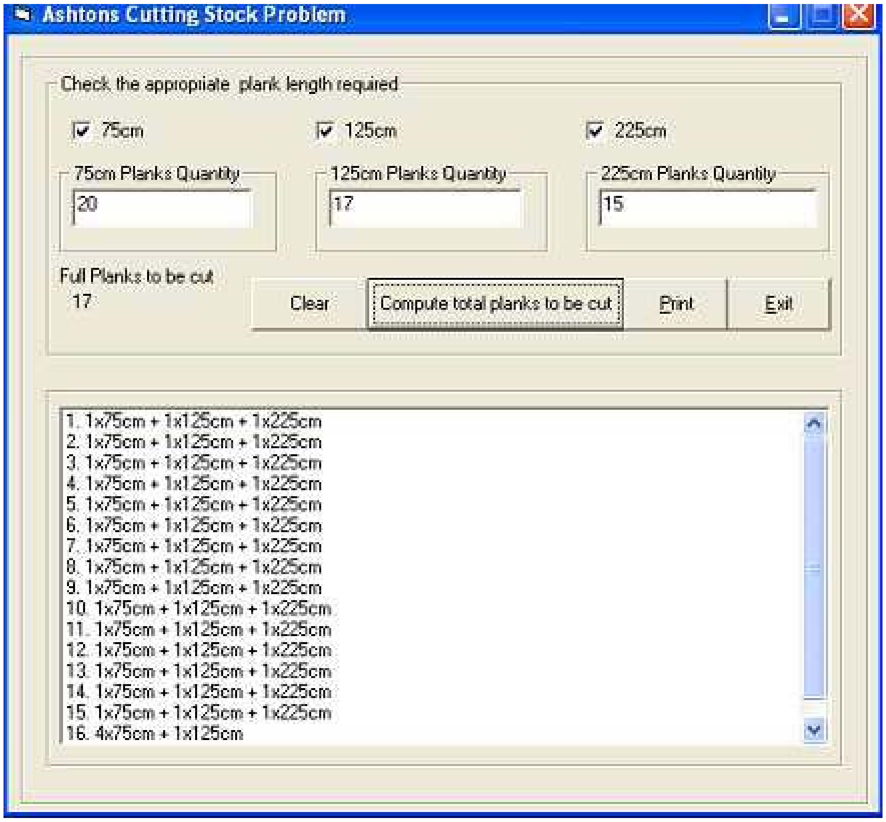

Figure2: Graphical User Interface displayed after in putting data

The results show that the people in the workshop will use 15 board of size 425cm to cut 15 pieces of each size and cut again 8 pieces of 75cm boards and then cut 2 pieces of 125cm boards and 2 boards of size 425cm. In this case there is no waste incurred. There are 17 full planks to be cut, these are 425cm boards. The pieces of wood to be kept in stock for future use in some cases are shown at the end of each cutting combination on the list box. To clear the text in each box just place clear button

SUMMARY, CONCLUSION AND RECOMMENDATION

SUMMARY

The research describes a column generation approach which decomposed the master problem into knapsack problem which was solved by the branch-and-bound procedure. The master problem was solved using the product form of the inverse. The product form of the inverse was used to solve the master problem to find the number of pieces of wood of length 425cm for each combination. It also gives the minimal waste incurred of 325cm from the optimal set x2, x5 and x6 after three iterations. The number of pieces of wood of 75cm, 125cm and 225cm cut was calculated and also the excess number of each length cut. This study showed that in order to minimize the waste incurred, the carpentry department should cut the recommended number of pieces of wood for each order required per week. The researcher came up with a graphical user interface using Visual Basic programming. The graphical user interface developed will be used by the carpentry department manager to meet demand for any order with minimum incurred waste. As supported by Woodnough (1991) a computer is a valuable instrument in the industry.

CONCLUSION

The study showed that the carpentry department is currently cutting many pieces of wood of length 425cm to meet the demand and they also incur a lot of waste. The carpentry department is currently cutting 20 pieces of wood of width 425cm, but this research shows that by applying column generation approach solved by the product form of the inverse, the carpentry department will cut 19 pieces of wood of width 425 cm with minimum waste incurred and have an excess of 2 boards of length 75 cm and 1 board of length 125cm in stock. The extra boards can be used to meet demand for the next order. And the graphical user interface will still be used for orders for each week. The printed combinations will be used by those people who work in the workshop.

RECOMMENDATIONS

For Ashtons to realise a significant minimal waste incurred besides using the GUI, they should concentrate on more rewarding pieces of wood cut with minimal waste, have an integrated, coordinated and advanced planning situation, examine carefully existing stock resource situation and past experiences as a basis for deciding on the new alternative enterprise and method to their situation best, identify clearly the various supply needs, sources, and demands for his alternative improvement plan, have a written document on the carpentry department plans on the cutting stock problem rather than planning mentally using trial and error and are recommended to use the graphical user interface for other orders in future to have minimum waste.

FURTHER RESEARCH

In solving real life problems, column generation approach can also be used to solve CSP involving cutting board of different lengths and widths, clothing material, steel sheets, and rolls of papers. Column generation approach can also be applied to create manpower schedules for workers at Ashtons.

REFERENCES

Dyckhoff, H., (1990). *A Typology of Cutting and Packing Problem*. European Journal of Operation Research, 44:145-159

Haessler, R. W., (1988) *Cutting Stock Problems And Solution procedures.*European Journal of Operation Research, 34:1460-1471

Hinxman,A. L., (1979). *The Trim-loss And Assortment Problems: A Survey : European to Fit Their Situation Bes*t. Journal of Operation Research, 5:8-18

Kalvelagen, E., (2002). *Column Generation with GAMS,* pp1-8, www.springelink.com/index/L9Q0681762TOMHGA.pdf

Kantorovich, L. V., (1960). *Mathematical Methods of Organizing And Planning Production: Management* Science, 6:366-422

Karelahti, J., (2002). *Solving the Cutting Stock Problem in the Steel Industry, pp1-8*

www.fp.mcs.anl.gov/otc/guid/casestudy/cutting/formation.html-6k-

Loren, A. N. L., (2000). *A Column Approch To Capacitated p-media Problem.Management Science,* 8:366-422

Schneider, D. I., (2004*). An introduction to Programming Using Visual, Basic 60 4thed* Pearson Education Ltd: London

Woodnough, M., (1991). *Fundamental of Mathematics*, Prentice Hall: London

Winston, A. E., (1994). *Introduction to Mathematical Programming: (Applicationand Algorithms) (3rd),* Duxbury Press : California